Hop Count Distance in Flooding-Based Mobile *Ad Hoc* Networks With High Node Density

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***Abstract*—In this paper, we analyze the behavior of flooding-based packet forwarding in a densely populated mobile *ad hoc* network. Specifically, we develop the probability distribution of hop count distance for a source–destination pair in the network, given that all nodes are roaming. The behavior of packet forwarding in such an environment is analogous to the ripples radiating from the source when one drops a stone into a pond. Due to node mobility, the number of hops traversed by each packet is not simply equal to the number of ripples from source to destination. In this paper, node mobility is represented as a growing circle centered at the destination node. The moving behavior of intermediate nodes can be ignored in a flooding-based *ad hoc* network with high node density since there is always a node available in the transmission direction to forward packets. The analytical model is validated via simulations. We further demonstrate that based on the proposed analytical model, one can estimate the flooding cost and search latency of target location discovery commonly used in most existing on-demand *ad hoc* routing protocols, and learn the impact of different flooding schemes on target discovery.**

***Index Terms*—Flooding, hop count distance, mobile *ad hoc* networks.**

I. Introduction

A MOBILE *ad hoc* network is a multihop wireless network in which each node plays both roles of a host and a router. Data packets are relayed via multiple hops, without the support of a fixed infrastructure. Mobile nodes in such a network may move arbitrarily, and the topology may vary with time.

The impact of node mobility on the performance of *ad hoc* networks has been intensively studied in recent years [1]–[17]. In [2]–[4], different mobility models are surveyed. Among them, the random waypoint model is the most commonly used mobility model. Its stochastic properties and node spatial distribution are studied in [5]–[7]. In [8]–[14], the link and path duration times of different mobility models are analyzed. In [15] and [16], the authors show that node mobility can improve the network capacity for mobile *ad hoc* networks. In [17], the effects of different mobility models on the performance of *ad hoc* routing protocols are evaluated.

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So far, less effort has been devoted to modeling the hop count distance between a source–destination (S–D) pair in mobile *ad hoc* networks. The importance of the analysis for hop count distance is obvious. For example, it can be used in physical location tracking or positioning in *ad hoc* and sensor networks [18]–[20], target location discovery in *ad hoc* on-demand routing protocols [21], and the estimation of the delivery ratio of packets with hop limits. In [22], the lower and upper bounds for the hop count estimation are provided under the assumption that the geographic location information of all nodes is known *a priori*. However, it does not consider node mobility, and thus, its applicability is limited.

In this paper, we model the hop count distance traversed by packets from source to destination in densely populated mobile *ad hoc* networks. The node movement is described by a growing circle centered at each node. The faster a node moves, the larger the circle. In our model, packets are forwarded via flooding, which is commonly used for target location discovery in *ad hoc* on-demand routing protocols. Based on the analytical model, we further evaluate different types of flooding mechanisms, including blind flooding (e.g., flooding described in [23]), two-tier flooding (e.g., dynamic source routing (DSR) [24]), and expansion-ring flooding (e.g., *ad hoc* on-demand distance vector (AODV) [25]), with respect to their flooding cost and target search latency for target location discovery. Compared to the work in [21] in which all nodes are assumed static, our model captures the impact of node mobility on the performance measurement. Such impact on cost and latency for target location discovery is not negligible, particularly when node speed is high. Besides, the destination node may move away or toward the source; therefore, the hop count is not fixed.

The rest of this paper is organized as follows. In Section II, the hop count distance is modeled, and the corresponding probability distribution is developed. In Section III, the analytical model is validated via simulations, and different types of flooding schemes are evaluated based on the model. Finally, this paper is concluded in Section IV.

II. Hop Count Distance for Flooding-Based Packet Forwarding

A. System Model and Assumptions

In this analysis, each node is equipped with the same transmission power and is continuously roaming in the network. Node mobility is described by a growing circle centered at the initial location of each node. The circles of fast-moving nodes grow faster than the circles of slow-moving nodes, which is similar to the concept proposed in [26]. We consider a densely

TABLE I
NOTATION USED IN THE ANALYSIS

a	Transmission range of each node
p	Packet size (bits)
c	Link speed (bps)
t_x	Packet transmission time, $t_x = \frac{p}{c}$
t_i	Random variable representing the processing delay at the ith node counting from the source on the multihop path
L	Random variable representing the initial distance between source and destination, with probability density function $f_L(l)$.
r_c	The growing rate of the circle centered at the destination node
H	Random variable representing the number of hops to be traversed for packet forwarding
$F_H(h_0)$	Cumulative distribution function of H, i.e., $\Pr(H \le h_0)$
$f_t(t_0)$	Probability density function of t_i
$f_t^*(s)$	Laplace transform of $f_t(t_0)$
$f_\Sigma(t_0)$	Probability density function of $\sum_{i=1}^{N-1} t_i$
$f_\Sigma^*(s)$	Laplace transform of $f_\Sigma(t_0)$
$f_L(l_0)$	Probability density function of L
$f_L^*(s)$	Laplace transform of $f_L(l_0)$
$P_H(k)$	Probability mass function of H, i.e., $\Pr(H = k)$

populated network, and thus, the isolated node problem [27] will not occur, and we ignore the border effect in the network so that nodes will not hit the boundary and bounce back in the analysis.

In this paper, we consider the number of hops traversed by packets, given that nodes are continuously roaming. Packets are forwarded by flooding. To focus on the hop count instance needed between source and destination, we will not consider interferences during packet transmissions [28] and the broadcast storm problem [29] in the network. The notation used in the analysis is summarized in Table I.

B. Multihop Packet Forwarding via Flooding

In this analysis, we consider a mobile *ad hoc* network with high node density, so that no matter to which direction the next hop is, there is always an intermediate node available at the edge of the transmission range to forward packets. Fig. 1 gives an example of our system, where a is the transmission range of each node. Interestingly, under such a high node density and flooding-based environment, the behavior of multihop packet forwarding is analogous to dropping a stone into a lake and generating ripples radiating from the source. At each hop, the traversed distance by the packet is equal to the transmission range a. We are particularly interested in the number of ripples between source and destination (i.e., the number of hops in the path), given that nodes are continuously roaming.

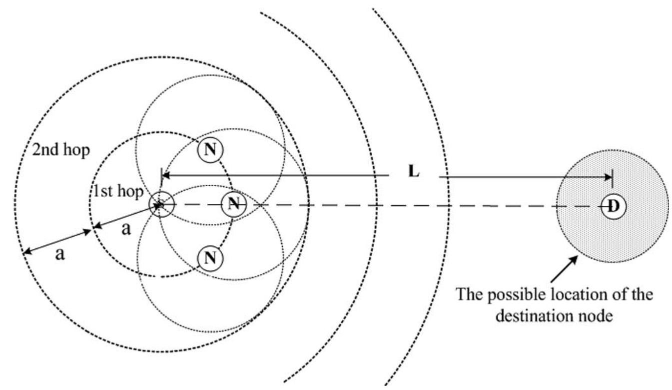

Fig. 1. Behavior of multihop packet forwarding via flooding in a dense mobile *ad hoc* network.

Since nodes may be moving around, the initial distance between source and destination, while important, may not reflect the actual number of hops traversed by packets for the S–D pair. If the destination node is moving toward the source, the distance will become shorter, and the final hop count will be less than the initial one, and vice versa. In short, the challenge in modeling hop count distance for mobile *ad hoc* networks stems mainly from the difficulty in predicting the direction to which the destination node is moving at any moment. What we ascertain is that if the average speed of the destination node is higher, the possible area the node may stay becomes larger, and vice versa. To capture this effect, we describe the node mobility as a growing circle centered at the initial location of each node and treat the circle coverage as the possible area the node may be located at any moment during packet forwarding. We focus only on the circle formed by the destination node. The moving behavior of intermediate nodes can be ignored in a flooding-based *ad hoc* network with high node density since there is always a node available in the transmission direction to forward packets.

C. Hop Count Distribution

Fig. 2 sketches three cases for the distance between an S–D pair after a packet traversal for N hops. In this figure, the larger circle, denoted by $Cov(S, N)$, represents the coverage traversed by flooded packets for N hops; the smaller shaded circle, denoted by $Cov(D, N)$, is the possible location of the destination node after N-hop packet traversal. Fig. 2(a) shows that the destination node stays in $Cov(S, N)$ when packet flooding starts, and it stays within this area after N-hop packet traversal. Fig. 2(b) depicts that initially, the destination node stays in $Cov(S, N)$, but it may move outside this area after the N-hop packet traversal. In Fig. 2(c), the destination node is outside $Cov(S, N)$ initially, but it may move into this area after the N-hop traversal.

To ensure that all packets can be relayed to the roaming destination node, the speed of the destination node is assumed to be lower than the forwarding rate. As such, the destination node can be caught up by flooded packets eventually, even though it may be moving away from the source. In other words, the circle formed by the possible location of the destination

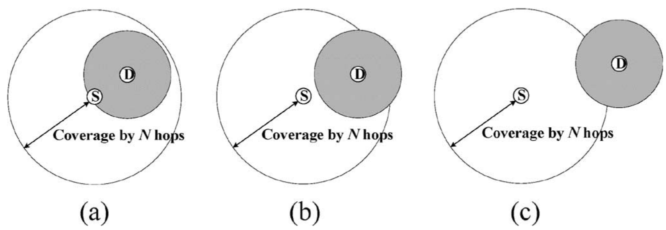

Fig. 2. Three cases for the relationship between $Cov(S, N)$ and $Cov(D, N)$. (a) Case 1. (b) Case 2. (c) Case 3.

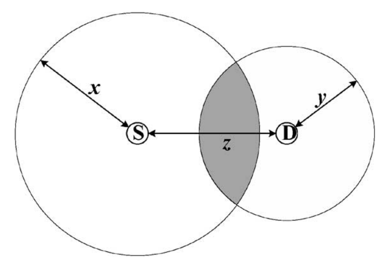

Fig. 3. Intersection area formed by circles with radii x and y.

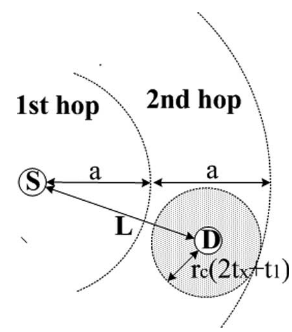

Fig. 4. Example of packet forwarding: $H \leq 2$.

node must be smaller than that formed by packet flooding, i.e., $Cov(D, N) < Cov(S, N)$. Therefore, for an N-hop packet traversal from the source, $r_c \cdot (N \cdot t_x + \sum_{i=1}^{N-1} t_i) \leq N \cdot a$ [i.e., $r_c \cdot (N \cdot t_x + \sum_{i=1}^{N-1} t_i)$ is the radius of $Cov(D, N)$, and Na is the radius of $Cov(S, N)$], yielding $\sum_{i=1}^{N-1} t_i \leq ((N \cdot a)/r_c) - N \cdot t_x$. Since node mobility is constrained by Na, the initial distance L of those nodes moving into (out of) the N-hop coverage, i.e., Case 3 (Case 2) in Fig. 2, should range from Na to $2Na$ (from 0 to Na). Therefore, the reachability requirement for an S–D pair can be defined as follows.

1) When the destination may move into the N-hop coverage

$$\sum_{i=1}^{N-1} t_i \leq \frac{N \cdot a}{r_c} - N \cdot t_x \quad N \cdot a < L < 2 \cdot N \cdot a. \tag{1}$$

2) When the destination moves out of the N-hop coverage

$$\sum_{i=1}^{N-1} t_i \leq \frac{N \cdot a}{r_c} - N \cdot t_x \quad 0 < L < N \cdot a. \tag{2}$$

The packet arrival probability is determined by the intersection area of $Cov(S, N)$ and $Cov(D, N)$. In Case 1, the arrival probability of at most N hops equals one since the small circle falls within the big circle entirely. In the other two cases, the arrival probabilities are proportional to the intersection area $Cov(S, N) \cap Cov(D, N)$. The ratio of this area $Cov(S, N) \cap Cov(D, N)$ to the circle $Cov(D, N)$ is defined as the reachability probability in this paper. Let x and y denote the radii of the two circles, as shown in Fig. 3. The circle with radius y corresponds to the possible location of the destination node, and that with radius x is the coverage via flooding; z is the initial distance between source and destination. Thus, the intersection area formed by the two circles can be calculated by

$$x^2 \cdot \cos^{-1}\left(\frac{x^2 + z^2 - y^2}{2xz}\right) + y^2 \cdot \cos^{-1}\left(\frac{y^2 + z^2 - x^2}{2yz}\right) - xz \cdot \sin\left(\cos^{-1}\left(\frac{x^2 + z^2 - y^2}{2xz}\right)\right)$$

if "$x - y \leq z \leq x + y$" is satisfied. For convenience, the intersection area can be approximated by $(1/2) \cdot \pi \cdot y(x + y - z)$, and the reachability probability is given by $(x + y - z)/(2 \cdot y)$.

Let H denote the hop count traversed by packets from source to destination in the network, and let $F_H(h_0)$ be the cumulative distribution function (cdf) of random variable H, i.e., $F_H(h_0) = \Pr(H \leq h_0)$. Let $F_H^{(i)}(N)$ denote the cdf of H in Case i. Since Cases 1, 2, and 3 are mutually exclusive, $F_H(N) = F_H^{(1)}(N) + F_H^{(2)}(N) + F_H^{(3)}(N)$. The hop count distribution $F_H^{(i)}(N), i = 1, 2, 3$, is developed as follows.

1) Case 1: Given the initial distance L, $a \geq L + r_c \cdot t_x$ must hold for $H \leq 1$. Thus

$$F_H^{(1)}(1) = \Pr(H \leq 1) = \Pr\left(L \leq \frac{ac - r_c p}{c}\right). \tag{3}$$

If the packet can reach the destination by at most two hops (as shown in Fig. 4), i.e., $H \leq 2$, we have $2a \geq L + r_c \cdot (2t_x + t_1)$, where t_1 is the processing delay of the first relay node in the path. Since $t_x = (p/c)$, we obtain $t_1 \leq (2ac - Lc - 2r_c p)/r_c c$. In addition, $t_1 \geq 0$, and thus, we have $(2ac - Lc - 2r_c p)/r_c c \geq 0$. Due to $r_c c \geq 0$, condition "$2ac - Lc - 2r_c p \geq 0$" must hold, i.e., $L \leq (2ac - 2r_c p)/c$. Thus

$$F_H^{(1)}(2) = \Pr(H \leq 2) = \int_0^{\frac{2ac - 2r_c p}{c}} P\left(t_1 \leq \frac{2ac - l_0 c - 2r_c p}{r_c c} \,\middle|\, L = l_0\right) f_L(l_0) dl_0 \tag{4}$$

where $f_L(l_0)$ is the probability density function (pdf) of L.

Similarly, the scenario that the packet traverses at most N hops occurs only when the following condition holds: $aN \geq L + r_c \cdot (N \cdot t_x + \sum_{i=1}^{N-1} t_i)$, where t_i is the processing delay of the ith relay node in the path, $i = 1, 2, \ldots, N-1$. Substituting $t_x = (p/c)$ into the inequality, we obtain $\sum_{i=1}^{N-1} t_i \leq (Nac - Lc - Nr_cp)/r_cc$. Since $0 \leq \sum_{i=1}^{N-1} t_i$, we have $L \leq (Nac - Nr_cp)/c$. Thus, the probability of at most N-hop traversal can be expressed by

$$F_H^{(1)}(N) = \Pr\{H \leq N\} = \int_0^{\frac{Nac-Nr_cp}{c}} P\left(\sum_{i=1}^{N-1} t_i \leq \frac{Nac-l_0c-Nr_cp}{r_cc} \middle| L=l_0\right) \times f_L(l_0)dl_0 = \int_0^{\frac{Nac-Nr_cp}{c}} \int_0^{\frac{Nac-l_0c-Nr_cp}{r_cc}} f_\Sigma(t_0)dt_0 \cdot f_L(l_0)dl_0 \quad (5)$$

where $f_\Sigma(t_0)$ is the pdf of $\sum_{i=1}^{N-1} t_i$.

Assume that $\{t_i : i = 1, 2, \ldots\}$ is a set of independent identically distributed random variables with pdf $f_t(t_0)$. Let $f_\Sigma^*(s)$ be the Laplace transform of $f_\Sigma(t_0)$. Thus, $f_\Sigma^*(s) = (f_t^*(s))^{N-1}$, where $f_t^*(s)$ is the Laplace transform of $f_t(t_0)$. For ease of derivation, the probability of at most N-hop traversal can be reexpressed by (6), shown at the bottom of the page.

2) Case 2: In Fig. 2(b), the destination node is in $Cov(S, N)$ when flooding begins, and as the packet traverses N hops, its location may be outside this coverage due to the node mobility. To describe such a scenario, the following two conditions must hold: 1) $0 \leq L \leq N \cdot a$ and 2) $L + r_c \cdot (N \cdot t_x + \sum_{i=1}^{N-1} t_i) \geq N \cdot a$. Together with the reachability requirement (i.e., $\sum_{i=1}^{N-1} t_i \leq ((N \cdot a)/r_c) - N \cdot t_x$ and $0 < L < N \cdot a$) and reachability probability mentioned earlier [i.e., $(N \cdot a + r_c(N \cdot t_x + t_0) - L)/(2 \cdot r_c(N \cdot t_x + t_0))$], the probability of at most N-hop traversal is expressed by

$$F_H^{(2)}(N) = \Pr(H \leq N) = \int_0^{Na} \int_{\frac{Na-l_0}{r_c}-N\cdot t_x}^{\frac{Na}{r_c}-N\cdot t_x} \frac{N \cdot a + r_c(N \cdot t_x + t_0) - l_0}{2 \cdot (N \cdot r_c \cdot t_x + r_c \cdot t_0)} \cdot f_\Sigma(t_0)dt_0 \cdot f_L(l_0)dl_0. \quad (7)$$

If $\{t_i : i = 1, 2, \ldots\}$ is a set of independent identically distributed random variables, the *probability* $F_H^{(2)}(N)$ of at most

$$\begin{aligned}
F_H^{(1)}(N) &= \int_0^{\frac{Nac-Nr_cp}{c}} \int_0^{\frac{Nac-l_0c-Nr_cp}{r_cc}} f_\Sigma(t_0)dt_0 \cdot f_L(l_0)dl_0 \\
&= \int_0^{\frac{Nac-Nr_cp}{c}} \int_0^{\frac{Nac-l_0c-Nr_cp}{r_cc}} \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} f_\Sigma^*(s)e^{st}ds\, dt_0 \cdot f_L(l_0)dl_0 \\
&= \int_0^{\frac{Nac-Nr_cp}{c}} \int_0^{\frac{Nac-l_0c-Nr_cp}{r_cc}} \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} [f_t^*(s)]^{N-1} e^{st}ds\, dt_0 \cdot f_L(l_0)dl_0 \\
&= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} [f_t^*(s)]^{N-1} \int_0^{\frac{Nac-Nr_cp}{c}} \int_0^{\frac{Nac-l_0c-Nr_cp}{r_cc}} e^{st}dt_0 \cdot f_L(l_0)dl_0\, ds \\
&= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} [f_t^*(s)]^{N-1} \cdot \int_0^{\frac{Nac-Nr_cp}{c}} \frac{1}{s}\left(e^{\frac{s(Nac-l_0c-Nr_cp)}{r_cc}} - 1\right) f_L(l_0)dl_0\, ds \\
&= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{[f_t^*(s)]^{N-1}}{s} \cdot \left[\int_0^{\frac{Nac-Nr_cp}{c}} \left(e^{\frac{s(Nac-Nr_cp)}{r_cc}} \cdot e^{-\frac{Sl_0}{r_c}} - 1\right) f_L(l_0)dl_0\right] ds \\
&= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{[f_t^*(s)]^{N-1}}{s} \cdot \left[e^{\frac{s(Nac-Nr_cp)}{r_cc}} \cdot \int_0^{\frac{Nac-Nr_cp}{c}} f_L(l_0) \cdot e^{-\frac{Sl_0}{r_c}}dl_0 - \int_0^{\frac{Nac-Nr_cp}{c}} f_L(l_0)dl_0\right] ds \qquad (6)
\end{aligned}$$

N-hop traversals can be reexpressed by

$$F_H^{(2)}(N) = \frac{1}{2}\cdot\frac{1}{2\pi j} \cdot \left[\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{(f_t^*(s))^{N-1}}{s} \int_0^{Na} \left(e^{\frac{sNa}{r_c}} - e^{\frac{s(Na-l_0)}{r_c}}\right) f_L(l_0) dl_0\, ds + \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{\int_{-\infty}^{s} (f_t^*(\omega))^{N-1} d\omega}{s} \times \int_0^{Na} \frac{Na-l_0}{r_c}\left(e^{\frac{sNa}{r_c}} - e^{\frac{s(Na-l_0)}{r_c}}\right) f_L(l_0) dl_0\, ds \right]. \quad (8)$$

3) Case 3: In Fig. 2(c), the destination node is located outside $Cov(S,N)$ when flooding begins, and its location may move into this coverage after N-hop traversal. In this case, the following two conditions must hold: 1) $N\cdot a \le L \le 2\cdot N\cdot a$, and 2) $L - r_c\cdot(N\cdot t_x + \sum_{i=1}^{N-1} t_i) \le N\cdot a$. Together with the reachability requirement (i.e., $\sum_{i=1}^{N-1} t_i \le ((N\cdot a)/r_c) - N\cdot t_x$ and $N\cdot a < L < 2\cdot N\cdot a$) and the reachability probability mentioned earlier [i.e., $(N\cdot a + r_c(N\cdot t_x + t_0) - L)/(2\cdot r_c(N\cdot t_x+t_0))$], the probability of at most N-hop traversals is expressed by

$$F_H^{(3)}(N) = \Pr(H \le N) = \int_{Na}^{2Na} \int_{\frac{l_0-Na}{r_c} - N\cdot t_x}^{\frac{Na}{r_c} - N\cdot t_x} \frac{N\cdot a + r_c(N\cdot t_x + t_0) + -l_0}{2\cdot r_c(N\cdot t_x + t_0)} \cdot f_\Sigma(t_0) dt_0 \cdot f_L(l_0) dl_0. \quad (9)$$

Again, if $\{t_i : i = 1,2,\ldots\}$ is a set of independent identically distributed random variables, the probability of at most N-hop traversals can be reexpressed by

$$F_H^{(3)}(N) = \frac{1}{2}\cdot\frac{1}{2\pi j} \cdot \left[\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{(f_t^*(s))^{N-1}}{s} \int_{Na}^{2Na} \left(e^{\frac{sNa}{r_c}} - e^{\frac{s(Na-l_0)}{r_c}}\right) f_L(l_0) dl_0\, ds + \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{\int_{-\infty}^{s} (f_t^*(\omega))^{N-1} d\omega}{s} \times \int_{Na}^{2Na} \frac{Na-l_0}{r_c}\left(e^{\frac{sNa}{r_c}} - e^{\frac{s(Na-l_0)}{r_c}}\right) f_L(l_0) dl_0\, ds \right]. \quad (10)$$

Since $F_H(N) = F_H^{(1)}(N) + F_H^{(2)}(N) + F_H^{(3)}(N)$, and $P_H(N)$, $N = 1,2,\ldots$, is the probability of exactly N-hop packet traversal, i.e., $P_H(N) = \Pr(H = N)$, we obtain

$$P_H(N) = \begin{cases} F_H(N), & \text{if } N = 1 \\ F_H(N) - F_H(N-1), & \text{if } N \ge 2 \\ 0, & \text{otherwise} \end{cases}.$$

III. Performance Evaluation

A. Analysis versus Simulation

We first validate the analytical model via simulations. Initially, 10 000 nodes are distributed in a disk of a radius of 100 units. Each node has a transmission range of 10 units and moves at a constant speed ($r_c = 0.5$ units/s) without changing its direction. The node in the center of the disk is selected as the source. The initial distance between an S–D pair and the processing delay at each intermediate node are exponentially distributed. To focus on the distribution of hop count distance in such a network, we do not consider channel errors and packet losses in our simulation.

The analytical curve is plotted based on the following derivation. Let t_i and L be exponentially distributed random variables with parameters λ_t and λ_l, respectively. Thus, $f_t(t_0) = \lambda_t e^{-\lambda_t t_0}$, $f_t^*(s) = \lambda_t/(s+\lambda_t)$; and $f_L(l_0) = \lambda_l e^{-\lambda_l l_0}$, $f_L^*(s) = \lambda_l/(s+\lambda_l)$. Since t_i is an exponential random variable, $\sum t_i$ will be a random variable with gamma distribution, and $f_\Sigma(t) = \lambda_t e^{\lambda_t t}\cdot((\lambda_t t)^{N-1}/(N-1)!)$.

Fig. 5 plots $F_H(N)$ with two different values of L. The figure shows that the analytical and simulation results fit very well. Next, we study the impact of different parameters on $P_H(N)$. Fig. 6 plots $P_H(N)$ as a function of L. We fix the following parameters: t_i is an exponential random variable with mean of 1 s, $a = 10$ units, and $r_c = 0.5$ units/s. For the curve of $H = 1$, the probability decreases as L increases, and vice versa, which matches our expectation. For the curves with $H > 1$, the probabilities increase as L increases, reach the maximum at some point, and then decrease. Consider $H = 2$, for example. When the initial distance between source and destination is short, the source and destination nodes are located within the transmission range of each other, and the packet only needs to traverse two hops. However, as L exceeds some threshold, the packet may need to traverse more than two hops, leading to a decrease in the probability of $H = 2$. Please note that the impacts of a and r_c on $P_H(N)$ are also very similar to the distance L. To save space, we will not include the figures in this paper.

B. Impact of Node Mobility on Hop Count for Packet Delivery

In this simulation, the following parameters are set: L and t_i are exponential random variables with mean values of 30 units and 1 s, respectively, $a = 10$ units, and $r_c = 0.5$ unit/s. Two nodes in the network are selected randomly as the source and the destination. Fig. 7 plots the simulation results for the average hop count and its standard deviation with different settings of node mobility. The average hop count increases as node mobility increases. This phenomenon can be explained

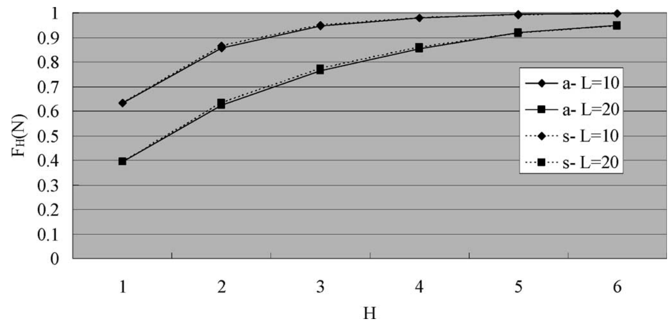

Fig. 5. Analytical and simulation results of $F_H(N)$.

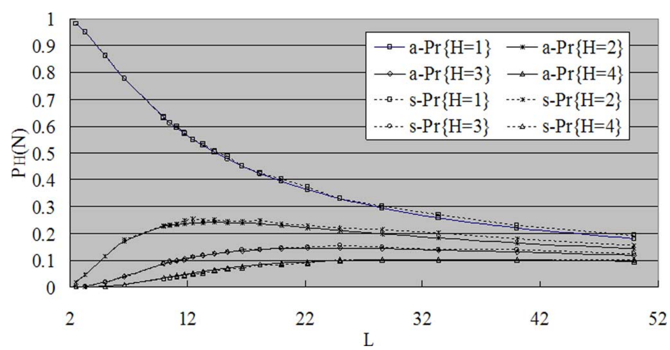

Fig. 6. Impact of L on $P_H(N)$.

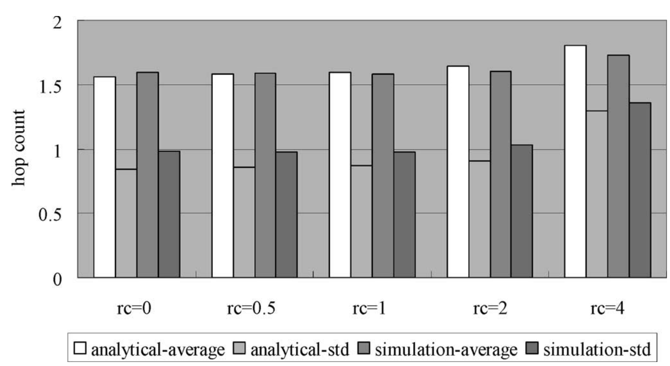

Fig. 7. Impact of node mobility on average hop count and its standard deviation.

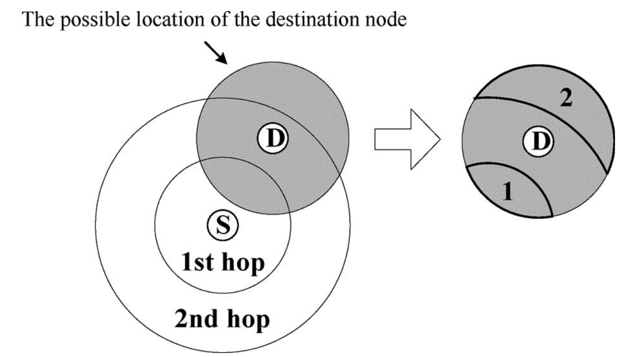

Fig. 8. Impact of node mobility on average hop count.

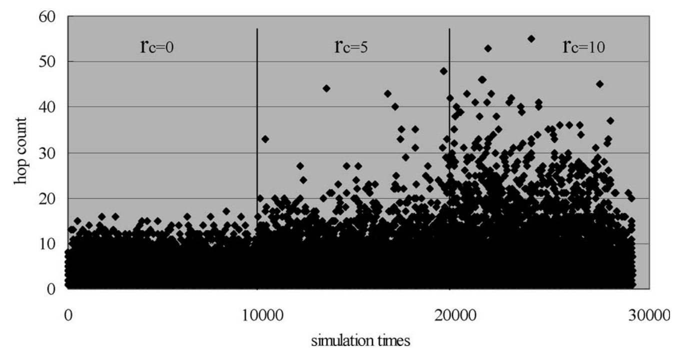

Fig. 9. Variation of hop counts for packet traversal.

by the example shown in Fig. 8, where the initial distance of the destination node D is two hops away from the source node S. The gray circle centered at D represents the possible area in which node D may move. It shows that Area 1, which represents node D moving toward the source, is smaller than Area 2, which indicates node D moving away from the source. In other words, the probability of node D moving away from S is larger than its coming closer to S. The differences in Areas 1 and 2 grow as the node mobility increases, leading to an increase in the average hop count for packet traversal.

To observe the effect of node mobility on the hop count variation for packet traversal, we measure the number of hops with three different speeds (i.e., $r_c = 0, 5, 10$), each running 10 000 simulation times. Fig. 9 shows that the fluctuation of hop counts traversed by packets becomes more significant as node mobility increases. Thus, the standard deviation of hop count for packet traversal increases as node mobility increases.

Fig. 7 also shows the analytical results based on our model in Section II. The average hop count is the expected value of random variable H. From the derived pdf, we obtain the analytical expected value and the standard deviation of H. The difference (which is very small) between the simulation and analytical results is caused by the simplification and assumption in the derivation process.

Next, we consider two scenarios with different t_i and r_c and compare the performance of our analytical model with existing models (e.g., [21]) that do not consider node mobility. In Fig. 10, the average hop count and its variation in our model (i.e., analysis with mobility) increase as the node mobility increases, which match the results generated by the simulations. The existing models that do not account for mobility, however, become inaccurate when node mobility exists, particularly in estimating the standard deviation of the hop count distance.

C. Performance of Different Flooding Schemes on Target Discovery

We evaluate the cost and latency of different flooding schemes for target destination discovery based on the proposed analytical model under various system parameters. Three different types of flooding schemes are considered in the evaluation.

1) Blind flooding: The entire network is flooded (e.g., [23]).
2) Two-tier flooding: The finite-hop neighbors are searched first. If the target is not found, the entire network is flooded. The searching packet for a target destination in DSR [24] is an example of two-tier flooding.

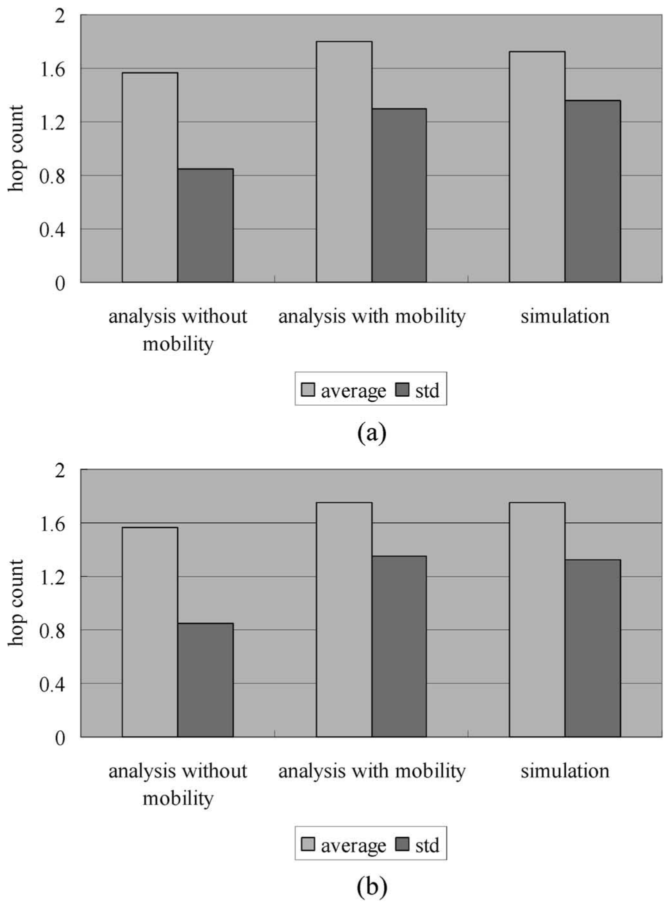

Fig. 10. Comparisons of analyses with and without node mobility consideration. (a) $t_i = 1$ and $r_c = 4$. (b) $t_i = 0.5$ and $r_c = 10$.

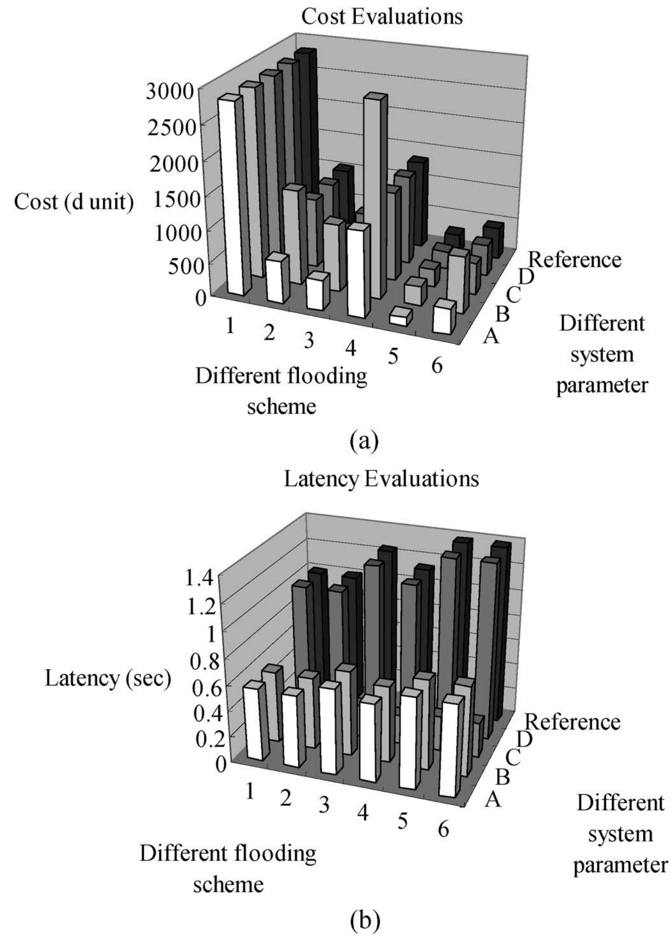

Fig. 11. Cost and latency of each scheme with different settings. (a) Cost. (b) Latency.

TABLE II
SYSTEM PARAMETER SETTINGS

	L (unit)	t_i (sec)	a (unit)	r_c (unit/sec)
A	6.67	1	10	0.5
B	10	1	15	0.5
C	10	0.2	10	0.5
D	10	1	10	0.1
Reference	10	1	10	0.5

3) Expansion-ring flooding: The source incrementally enlarges the searching range from an initial value to a predefined threshold. If the target is still not found, the entire network is flooded. The searching packet for a target destination in AODV [25] is an example of expansion-ring flooding.

We consider six different schemes. Scheme 1 is blind flooding. Schemes 2–4 are two-tier flooding with i-hop initial search (i.e., different settings to limit the flooding range of the initial packet); $i = 1, 2,$ and 3 for Schemes 2–4, respectively. Schemes 5 and 6 are expansion-ring flooding with j-hop initial search; $j = 1$ and 2 for Schemes 5 and 6, respectively. There are no predefined thresholds for Schemes 5 and 6. The eligible nodes for a k-hop initial search are those located in the area with radius $(k-1)a$, $k = 1, 2, \ldots$, but not including those exactly k hops away, from the source. Thus, the set of nodes eligible for a k-hop initial search includes the nodes located in the area

TABLE III
COST SAVING (CS) AND LATENCY REDUCTION (LR) (IN PERCENTAGE)

Scheme	1		2		3	
%	CS	LR	CS	LR	CS	LR
A	0	44	39.6	44	33.1	46.8
B	0	44.3	-36.5	44.3	-49.2	47.2
C	0	80.1	0	80.1	2.2	80.2
D	0	0.5	0	0.5	2.2	1
Scheme	4		5		6	
%	CS	LR	CS	LR	CS	LR
A	4.9	47.2	52	49.1	20.7	49.1
B	-114	47.6	-6.5	49.6	-77.6	49.6
C	0.5	80.2	1.9	80.3	1.1	80.3
D	0.5	1.1	1.9	1.4	1.1	1.4

of $(k-1)^2\pi a^2 d$, $k = 1, 2, \ldots$. Note that since a dense *ad hoc* network is considered, $d \gg 1$.

We evaluate the flooding cost and search latency of a target location for each scheme, where the cost is defined as the number of times a packet is broadcast by each scheme, and the latency is the round trip delay between source and destination. The cost and latency for each type of flooding schemes in a network with diameter k hops are defined as follows.

1) Blind flooding: Since the packet is flooded to the entire network, the cost of blind flooding, which is denoted by C_k, is equal to the number of nodes located in $(k-1)$ hops, and the latency is equal to $\sum_{l=1}^{k} P_l \cdot 2lT$, where P_l

is the probability that the target is located at the lth hop, and T is the per-hop latency.

2) Two-tier flooding with initial j-hop search: The cost is $(\sum_{i=1}^{j} P_i) \cdot C_j + (\sum_{l=j+1}^{k} P_l) \cdot C_k$, and the latency is $\sum_{i=1}^{j} P_i \cdot 2iT + \sum_{l=j+1}^{k} P_l \cdot (2lT + 2jT)$.
3) Expansion-ring flooding with initial j-hop search: The cost is $(\sum_{i=1}^{j} P_i) \cdot C_j + \sum_{l=j+1}^{k} P_l \cdot C_l$, and the latency is $\sum_{i=1}^{j} P_i \cdot 2iT + \sum_{l=j+1}^{k} P_l \cdot (2lT + \sum_{k=1}^{l-1} 2kT)$.

We consider four sets of parameter settings listed in Table II for performance evaluation. The "Reference" row indicates the baseline of comparison for different flooding schemes. Compared to the baseline, Row A has 33% reduction in L (i.e., initial distance for an S–D pair), Row B has 50% increase in a (transmission range of a node), Row C has 80% reduction in t_i (i.e., processing delay of an intermediate node), and Row D has 80% reduction in r_c (i.e., node mobility).

Fig. 11 shows the flooding cost and search latency of each scheme with different network topologies listed in Table II. Table III lists the corresponding cost saving and latency reduction for each flooding scheme compared with the baseline setting. We have the following observations from Fig. 11 and Table III.

1) Tradeoff Between Flooding Cost and Search Latency: Fig. 11 shows the tradeoff between flooding cost and search latency for different schemes. For blind flooding, the packet is broadcast to the entire network and never retransmitted. Thus, it has the highest flooding cost but the lowest latency. For two-tier flooding, the packet is hop-limited in its first probing, followed by blind flooding if the probing fails. As a result, it can significantly reduce the cost without a dramatic increase in the latency. For expansion-ring flooding, the flooding range increases incrementally. Thus, it has the lowest cost but at the expense of higher latency due to multiple refloodings. In short, based on the proposed model, as long as the network parameters are given, we can estimate the flooding cost and latency with different flooding strategies. With different requirements, the corresponding appropriate flooding scheme can be determined.

2) Impact of Different Parameters on Each Flooding Scheme: Table III shows that only the initial distance L and the transmission range a of each node can differentiate the performance of different schemes. The other two parameters affect the performance of all schemes in a similar way. We examine the impacts of a change in L (i.e., Row A) and a change in a (i.e., Row B) on the flooding cost of all schemes. Blind flooding is immune to the changes in the values of L and a, because it always floods to the entire network. For the two-tier and the expansion-ring schemes, the observations are that 1) a reduction in L can lower (and an increase in a can raise) the flooding cost of both types of schemes and 2) a smaller initial search range saves more flooding cost (see Columns 2 and 5 for example, which corresponds to the case of one-hop initial search). Such observations match our intuition.

IV. Conclusion

In this paper, we model the hop count distance for flooding-based mobile *ad hoc* networks with high node density. To make our analytical results generally applicable, we do not assume a particular mobility model. Instead, the node mobility is captured by a circle centered at the initial location of the destination node. The behavior of packet flooding in a dense mobile *ad hoc* network is analogous to dropping a stone into a lake and then counting the number of ripples traversed by a packet for an S–D pair. Due to node mobility, the number of ripples is not equal to the hop count distance for packet traversal. We then develop the probability distribution function of hop count distance for an S–D pair, given that nodes are continuously roaming. Based on the analytical model, the flooding cost and search latency of different flooding schemes for target location discovery can also be obtained.

The analytical model of the hop count distance can be used to examine the performance of various mechanisms in mobile *ad hoc* networks or sensor networks. For example, it can be used to evaluate the performance of various on-demand *ad hoc* routing protocols on target location discovery, to analyze the performance of hop-counting techniques in target localization in sensor networks [18]–[20], and to estimate the delivery ratio of packets that are transmitted with hop limits. In this paper, we only demonstrate its applicability to target location discovery for on-demand routing protocols in mobile *ad hoc* networks. Compared to existing work on this subject (e.g., [21]), our model considers the effect of node mobility on the performance evaluation and, thus, provides more accurate results and more insights on target location discovery for mobile *ad hoc* networks.

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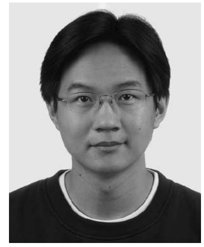

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